An internal-multiple *elimination* algorithm for all first-order internal multiples for a 1D earth

Yanglei Zou, Arthur B. Weglein, M-OSRP/Physics Dept./University of Houston

SUMMARY

The ISS (Inverse-Scattering-Series) internal-multiple attenuation algorithm (Araújo et al. (1994),Weglein et al. (1997) and Weglein et al. (2003)) can predict the correct time and approximate amplitude for all first-order internal multiples without any subsurface information. When combined with an energy minimization adaptive subtraction, the ISS internal multiple attenuation algorithm can effectively remove internal multiples when the primaries and internal multiples are separated, and not overlapping or proximal. One of issues that the adaptive subtraction is addressing is the difference between the amplitude of the internal multiple and the approximate amplitude of the attenuation algorithm prediction. However, under certain circumstances, both offshore and onshore, internal multiples are often proximal to or interfering with primaries and the criteria of adaptive subtraction may fail, since the energy can increase when e.g., a multiple is removed from an interfering primary. Therefore, in these situations, the task of removing internal multiples without damaging primaries becomes more challenging and subtle and currently beyond the collective capability of the petroleum industry. Weglein (2014) proposed a three-pronged strategy for providing an effective response to this pressing and prioritized challenge. One part of the strategy is to develop an internal-multiple elimination algorithm that can predict both the correct amplitude and correct time for all internal multiples. In this paper, we provide an ISS internal-multiple elimination algorithm for all first-order internal multiples generated from all reflectors in a 1D earth and provide an example from an elastic synthetic data that shows the value provided by the new algorithm in comparison with the value provided by the internal multiple attenuation algorithm.

INTRODUCTION

The ISS (Inverse-Scattering-Series) allows all seismic processing objectives, such as free-surface-multiple removal and internal-multiple removal to be achieved directly in terms of data, without any estimation of the earth's properties. For internal-multiple removal, the ISS internal-multiple attenuation algorithm can predict the correct time and approximate and well-understood amplitude for all first-order internal multiples generated from all reflectors, at once, without any subsurface information. If the events in the data are isolated, the energy minimization adaptive subtraction can fix the gap between the attenuation algorithm prediction and the internal multiples plus, e.g., all factors that are outside the assumed physics of the subsurface and acquisition. However, in certain situations, events often interfere with each other in both on-shore and off-shore seismic data. In these cases, the criteria of energy minimization adaptive subtraction may fail and completely removing internal multiples becomes more challenging and beyond the current capability of the petroleum industry.

For dealing with this challenging problem, Weglein (2014) proposed a three-pronged strategy including

1. Develop the ISS prerequisites for predicting the reference wave field and to produce de-ghosted data.
2. Develop internal-multiple elimination algorithms from ISS.
3. Develop a replacement for the energy-minimization criteria for adaptive subtraction.

To achieve the second part of the strategy, that is, to upgrade the ISS internal-multiple attenuation algorithm to elimination algorithm, the strengths and limitations of the ISS internal-multiple attenuation algorithm are noted and reviewed. The ISS internal-multiple attenuation algorithm always attenuates all first-order internal multiples from all reflectors at once, automatically and without any subsurface information. That is a tremendous strength, and is a constant and holds independent of the circumstances and complexity of the geology and the play. However, there are two well-understood limitations of this ISS internal-multiple attenuation algorithm

1. It may generate spurious events due to internal multiples treated as subevents.
2. It is an attenuation algorithm not an elimination algorithm.

The first item is a shortcoming of the leading order term (the attenuation algorithm), when taken in isolation, but is not an issue for the entire ISS internal-multiple removal capability. It is anticipated by the ISS and higher order ISS internal multiple terms exist to precisely remove that issue of spurious events prediction. When taken together with the higher order terms, the ISS internal multiple removal algorithm no longer experiences spurious events prediction. Ma et al. (2012) , H. Liang and Weglein (2012) and Ma and Weglein (2014) provided those higher order terms for spurious events removal.

In a similar way, there are higher order ISS internal multiple terms that provide the elimination of internal multiples when taken together with the leading order attenuation term. There are early discussions in Ramírez (2007) and Wilberth Herrera and Weglein (2012) find higher order terms in ISS that can eliminate all first-order internal multiples generated at the shallowest reflector for 1D normal incidence spike plane wave. The next step, elimination of all first-order internal-multiples generated from all reflectors, is a very challenging problem even in a 1D earth. In a model with several reflectors, there is a set of internal multiples generated by each reflector in the data, and for different set of internal multiples, the amplitude difference between attenuation algorithm prediction and the real internal multiples is different. This elimination algorithm must have the capability to remove all the amplitude differences between attenuation algorithm prediction and the real internal multiples for all sets of internal multiples. While the activity of finding higher order terms in ISS that can completely eliminate all internal multiples is undertaken, we derived an elimination algorithm from an alternative approach, i.e. using reverse engineering method as a guide and a way to understand the internal multiple elimination machinery in the ISS. This elimination algorithm can predict both correct time and amplitude of all first-order internal-multiples generated from all reflectors in a 1D earth. And it is closely related to ISS and preserves the following advantages of ISS attenuation

1. It only needs data, does not require any subsurface information.
2. It provides the capability to remove all first-order internal multiples without stripping.

And this elimination algorithm derived by using reverse engineering method is model type dependent. (The ISS internal-multiple attenuation algorithm is model type independent.)

ISS INTERNAL-MULTIPLE ATTENUATION ALGORITHM AND ATTENUATION FACTOR FOR A 1D NORMAL INCIDENCE SPIKE PLANE WAVE

First, we will give an introduction of the ISS internal-multiple attenuation algorithm before we introduce the internal-multiple elimination algorithm. The ISS internal-multiple attenuation algorithm is first given by Araújo et al. (1994) Weglein et al. (1997). The 1D normal incidence version of the algorithm is presented as follows:

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \times \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z''). \quad (1)$$

Where $b_1(z)$ is the water speed migration of the data of a 1D normal incidence spike plane wave. ε_1 and ε_2 are two small positive numbers introduced to avoid self interactions. $b_3^{IM}(k)$ is the predicted internal multiples in the vertical wavenumber domain. This equation can predict the correct time and approximate amplitude of all first-order internal multiples.

The procedure of predicting a first-order internal multiple generated at the shallowest reflector is shown in figure 1. The ISS internal-multiple attenuation algorithm automatically uses three primaries in the data to predict a first-order internal multiple. (Note that this algorithm is model type independent and it takes account all possible combinations of primaries that can predict internal multiples.) From this figure we can see, every sub event on the left hand side experiences several phenomena making its way down to the earth then back to the receiver. When compared with the internal multiple on the right hand side, the events on the left hand side have extra transmission coefficients as shown in red. Multiplying all those extra transmission coefficients, we get the AF (attenuation factor) - $T_{01}T_{10}$ for this first-order internal multiple generated at the shallowest reflector. And all first-order internal multiples generated at the shallowest reflector have the same attenuation factor.

Figure 2 shows the procedure of predicting a first-order internal multiple generated at the next shallowest reflector. In this example, the attenuation factor is $(T_{01}T_{10})^2(T_{12}T_{21})$.

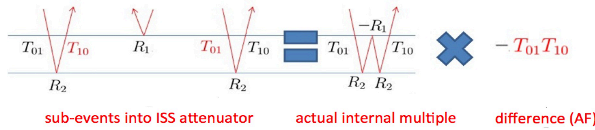

Figure 1: an example of the attenuation factor of a first-order internal multiple generated at the shallowest reflector, notice that all red terms are extra transmission coefficients

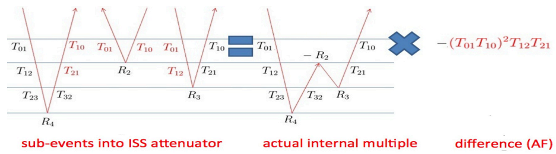

Figure 2: an example of the attenuation factor of a first-order internal multiple generated at the next shallowest reflector, notice that all red terms are extra transmission coefficients

The attenuation factor for predicting a multiple generated by the i^{th} reflector, AF_j, is given by the following:

$$AF_j = \begin{cases} T_{0,1}T_{1,0} \quad (for\ j=1) \\ \prod_{i=1}^{j-1}(T_{i-1,i}^2 T_{i,i-1}^2)T_{j,j-1}T_{j-1,j} \quad (for\ 1<j<J) \end{cases} \quad (2)$$

The attenuation factor AF_j can also be performed by using reflection coefficients:

$$AF_j = \begin{cases} 1-R_1^2 \quad (for\ j=1) \\ (1-R_1^2)^2(1-R_2^2)^2\cdots(1-R_j^2) \quad (for\ 1<j<J) \end{cases} \quad (3)$$

The subscript j represents the generating reflector, and J is the total number of interfaces in the model. The interfaces are numbered starting with the shallowest location. The attenuation factor is directly related to the trajectory of the events, which forms the prediction of the internal multiple.

ISS INTERNAL-MULTIPLE ELIMINATION ALGORITHM FOR A 1D NORMAL INCIDENCE SPIKE PLANE WAVE

The discussion above demonstrates that all first-order internal multiples generated at the same reflector have the same attenuation factor. We can observe that the attenuation factor contains all transmission coefficients from the shallowest reflector down to the reflector generating the multiple. And from the examples(shown in figure 1 and 2) we can observe that the middle event contains all the information about those transmission coefficients. Following early discussions and work in

Ramírez (2007) and Wilberth Herrera and Weglein (2012), our idea is to modify the middle term in the attenuation algorithm to remove the attenuation factor and make the attenuation algorithm an elimination algorithm. That is from

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \times \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'') \tag{4}$$

to

$$b_E^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} F[b_1(z')] \times \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'') \tag{5}$$

where $F[b_1(z)]$ is an intermediate function we need to discover.

By introducing another intermediate function g(z) in which the amplitude of each event corresponds to a reflection coefficient, we discovered a way to construct $F[b_1(z)]$ by using $b_1(z)$ and g(z). After that, we find an integral equation about $b_1(z)$ and g(z). The $F[b_1(z)]$ is first proposed in Zou and Weglein (2013):

$$F[b_1(z) = \frac{b_1(z)}{[1-(\int_{z-\varepsilon}^{z+\varepsilon} dz' g(z'))^2][1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')]^2} \tag{6}$$

$$g(z) = \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')} \tag{7}$$

To derive the $F[b_1(z)]$ from $b_1(z)$, g(z) must first be solved in equation (7). Thereafter, g(z) is integrated into equation (6). Now we will show one way to solve these equations. By iterating $g(z)$ in (7), we can get more accurate approximation. Substitute more accurate approximations of $g(z)$ into $F[b_1(z)]$, we will achieve or obtain higher orders of approximation of the elimination algorithm which can predict correct amplitude of first-order internal multiples generated at deeper reflectors.

ISS INTERNAL-MULTIPLE ELIMINATION ALGORITHM FOR 1D PRESTACK DATA

The 1D prestack data is more complicated than 1D normal incidence data in two aspects: (1) The 1D prestack data has one more variable x (or k in wavenumber domain); (2)The reflection coefficients become angle dependent. Fortunately, following discussions and examples in Zou and Weglein (2014), we discovered that the same elimination algorithm scheme is still valid for 1D pre-stack data. Below shows the 1D prestack internal-multiple elimination algorithm, where $b_1(k,z)$ is the water speed uncollapsed Stolt migration of the data; $b_E^{IM}(k,2q)$ is the elimination algorithm prediction in wavenumber domain; $F[b_1(k,z)]$ and $g(k,z)$ are two intermediate functions and they are related by equation (9) and (10)

$$b_E^{IM}(k,2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k,z) \int_{-\infty}^{z-\varepsilon_1} dz' e^{-2iqz'} F[b_1(k,z')] \times \int_{z'+\varepsilon_2}^{\infty} dz'' e^{2iqz''} b_1(k,z'') \tag{8}$$

$$F[b_1(k,z)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times \frac{e^{-iq'z} e^{iq'z'} b_1(k,z')}{[1-\int_{-\infty}^{z'-\varepsilon} dz'' b_1(k,z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k,z''') e^{-iq'z'''}]^2} \times \frac{1}{1-|\int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(k,z'') e^{iq'z''}|^2} \tag{9}$$

$$g(k,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times \frac{e^{-iq'z} e^{iq'z'} b_1(k,z')}{1-\int_{-\infty}^{z'-\varepsilon} dz'' b_1(k,z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k,z''') e^{-iq'z'''}} \tag{10}$$

NUMERICAL TESTS FOR SYNTHETIC ELASTIC PP DATA

We test the 1D pre-stack internal multiple elimination algorithm for an four-reflector elastic model shown in figure 3. Figure 4 shows the PP data generated from this model by reflectivity method. Figure 5 and figure 6 show a section(2.8s-3.1s) of the data and attenuation and elimination prediction results.

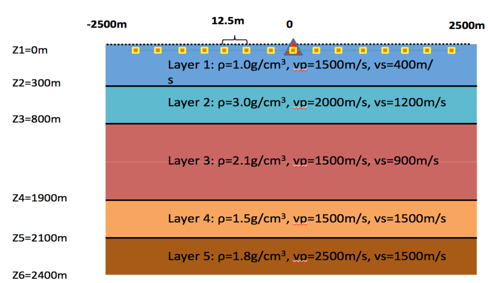

Figure 3: model

The left picture in figure 5 shows a section in the input data. In this section ,there are 3 major events interfering with each other: a converted P primary, an internal multiple generated from the first reflector and another internal multiple generated from the third reflector. The middle picture in figure 5 shows the attenuation algorithm predicted internal multiples,

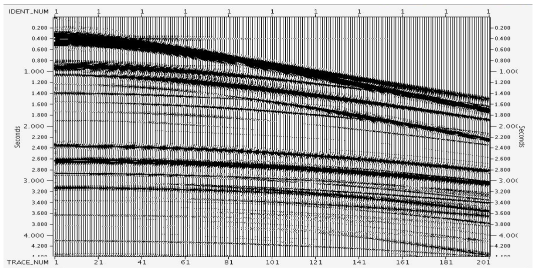

Figure 4: PP data

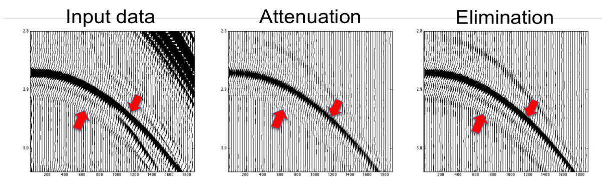

Figure 5: A section of the input data and prediction. Left: input data. Middle: attenuation algorithm prediction. Right:elimination algorithm prediction.

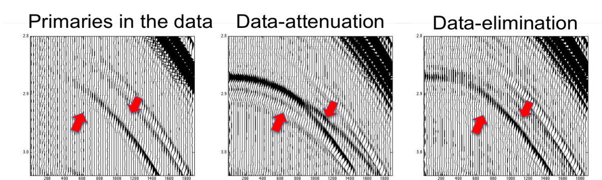

Figure 6: Left: primaries in the input data. Middle: data after internal multiples being attenuated. Right:data after internal multiples being eliminated.

it clearly shows the predicted internal multiples have correct time and approximate amplitude. The right picture in figure 5 shows the elimination algorithm prediction, the time is correct and the amplitude is more accurate. The left picture in figure 6 shows the primaries in the data. (Because it is a synthetic test, we can generate only the primaries and use them as a benchmark.) The middle picture in figure 6 shows the result by subtracting the attenuation algorithm prediction from the data. The internal multiples has been reduced, but there still remains residues. The right picture in figure 6 shows the result by subtracting the elimination algorithm prediction from the data. We can see that the multiples has been (almost) completed eliminated and the primary is recovered. (Note that there is still some small residues in the near offset due to the inaccuracy of numerical Hankel transform.)

A LIMITATION OF THIS ELIMINATION ALGORITHM AND AN ALGORITHM TO ADDRESS THE LIMITATION

There is a limitation of this elimination algorithm, that is, the primaries in the reflection data that enters the algorithm provides that elimination capability, automatically without requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, may alter the amplitude and need assist to completely eliminate the internal multiples. That is a limitation of this elimination algorithm. To deal with this limitation, we can first use ISS internal multiple attenuation algorithm prediction (b_3) to attenuate the internal multiples in the data (b_1) and then put the subtracted result, i.e., $b_1 + b_3$, which contains primaries and attenuated internal multiples, into the ISS internal multiple elimination algorithm to predict internal multiples with more accurate amplitude. (Note that in the numerical test, this limitation has very small affect on the prediction, thus we do not need to consider addressing this limitation in this test. However, for certain situation, we need to consider this limitation.)

CONCLUSION

The ISS internal multiple elimination algorithm is a part of the three-pronged strategy which is especially relevant and provide value when primaries and internal multiples are proximal to and/or interfere with each other in both on-shore and off-shore data. While the activity of finding higher order terms in ISS that can completely eliminate all internal multiples in multi-D is undertaken, we derive an elimination algorithm from a reverse engineering approach and use it as a guide and a way to understand the internal multiple elimination machinery in the ISS. In this paper, we discussed the 1D pre-stack ISS internal multiple elimination algorithm for all first-order internal multiples from all reflectors and tested this algorithm with an elastic synthetic PP data. The result shows that this ISS internal multiple elimination algorithm can predict more accurate amplitude of the internal multiples than the attenuation algorithm.

This reverse engineering approach launched by understanding the properties of the ISS attenuation algorithm, and in 1D, reverse engineering a solution, was based on the input being primaries. The primaries in the reflection data that enters the algorithm provide that elimination capability, automatically without our requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, will not be helpful in this elimination scheme. This can be a limitation and can sometimes have significant affect on the prediction. We also provide an algorithm to address this limitation in the last section.

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